Application of the boundary face method for thermal analysis in dam construction*

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ABSTRACT: *This paper applies the boundary face method (BFM) to solve both steady-state and unsteady-state heat conduction problems in the dam construction. Instead of employing the isoparametric elements as in the boundary and finite element methods, the BFM uses directly geometric models from CAD packages to avoid geometric errors for any complex structures. Moreover, the variable approximations and the boundary integration are all performed in the parametric space of the boundary surfaces, which means that the elements and integral patches are defined in the parametric space. A numerical example of steady-state analysis presented in this paper is a concrete dam structure which consists of several blocks. The example of unsteady-state analysis, which concerns a single block with several holes in it, has also been presented to illustrate the validity and the efficiency of the proposed method.*

KEYWORDS: Boundary face method; steady-state heat conduction; unsteady-state heat conduction; dam construction; small holes.

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1 INTRODUCTION

Thermal effects are usually related to the damage to structures made of concrete including bridges and dams. Thus prediction of thermal and stress is of great importance to the design of concrete structures. In the procedure of dam construction, the heat conduction is of great importance to its design and construction. During cooling of the concrete dam, the high thermal gradient, which is caused by the hydration of cement, usually leads to significant thermal tensile stresses. These stresses are the main factors of the cracks inside the dam. Even the quantity of cement in the concrete, which is used in the dam construction, is much less than that of conventional concrete.

Since the massive dam is usually of very large size, the computation scale of thermal analysis by finite element method (FEM) implementation will exceed hundreds of millions degrees, and this level of analysis cannot be performed in micro-computers.

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Because of the computational scale limitation, the overall analysis on the dam body by FEM is usually unavailable. In practical applications, many special elements including multi-layer element are usually employed (Zhu & Xu, 1996; Chen et al, 2001). The employment of these special elements, however, will inevitably introduce errors.

In this paper, we introduce a boundary integral equation (BIE) based method (Brebbia, 1978; Liu, 1998), which is called the boundary face method (BFM) (Zhou et al, 2011; 2012, Qin et al, 2010; Zhang et al, 2009), to perform the thermal analysis on the dam body. In this method, only boundary discretisation is required, thus the number of degrees is one order less than that in the FEM. To circumvent the full matrix problem in the BFM, a fast scheme which combines the adaptive cross approximation technology (Kurz et al, 2002) and hierarchical matrix (H-matrix) (Hackbusch, 1999) is applied to construct the corresponding solver. A dam containing 13 blocks is considered in the first numerical implementation. This example illustrates the accuracy and validity of the method.

In order to avoid the cracks caused by the large temperature gradient, cooling pipes are usually laid in each layer of the concrete to conduct the heat through the water flow in practical construction procedure. In most cases, these pipes are of tiny sizes. In most FEM implementations, however, a large number of additional grids, which further increase the computational scale of the considered problem, are required to capture these tiny features. In practical implementations, these pipes are usually simplified with some empirical assumptions (Zhu, 2010). These assumptions will inevitably introduce errors especially in the places where the gradient of temperature is large and where cracks most likely occur.

As a preliminary study, we have also implemented a transient thermal analysis by BFM in one block of the dam containing many curved cooling pipes. In this analysis, a convolution quadrature scheme is employed to deal with the time issues in transient problems. In the convolution scheme, the effect of temperature from all the past steps is expressed by a series of time convolution instead of by the pseudo-initial temperature computed in the last step. In the traditional scheme, a large number of grids is required to present these tiny holes, thus the computational scale will be largely increased. In this paper, with employing a special type of elements, the number of nodes for this problem remains the same level as that without considering the pipes. A transient analysis on a block which contains several cylindrical holes is performed as the second illustration example in this study. In this example, comparison study with the FEM based software showing that the efficiency of the proposed method for the transient heat conduction problem.

2 BOUNDARY FACE METHOD FOR 3D STEADY HEAT CONDUCTION

2.1 Boundary integral equation for 3D steady heat conduction

The steady heat conduction problem in three dimensions can be mathematically described as:

$$\begin{aligned} &u_{,ii} = 0, && \forall x \in \Omega \\ &u = \bar{u}, && \forall x \in \Gamma_u \\ &u_{,i} n_i \equiv q = \bar{q}, && \forall x \in \Gamma_q \end{aligned} \tag{1}$$

where Ω is the corresponding domain which is enclosed by $\Gamma = \Gamma_u \bigcup \Gamma_q$. On the essential boundary Γ_u and the flux boundary Γ_q, we impose the boundary condition $\bar{u}$ and $\bar{q}$, which denote the prescribed temperature and the normal flux, respectively. n with components n_i, i = 1, 2, 3 is the outward normal on the boundary Γ.

The problem can be converted into an equivalent BIE which is described as the following formulation:

$$\int_\Gamma (u(\mathbf{s}) - u(\mathbf{y})) q^s(\mathbf{s},\mathbf{y}) d\Gamma = \int_\Gamma q(\mathbf{s}) u^s(\mathbf{s},\mathbf{y}) d\Gamma \tag{2}$$

In this formula, **y** and **s** stand for the field point and source point, respectively. $q = \partial u / \partial n$ is the boundary normal flux. $u^s(\mathbf{s}, \mathbf{y})$ and $q^s(\mathbf{s}, \mathbf{y})$ are the corresponding fundamental solutions which satisfy:

$$u^s_{,ii} = \delta(y, s), \ \ \forall s \in \Omega \tag{3}$$

and

$$q^s = \partial u^s / \partial n(s) \tag{4}$$

For 3D problems”

$$u^s(\mathbf{s},\mathbf{y}) = \frac{1}{4\pi r(\mathbf{s},\mathbf{y})} \tag{5}$$

where r is the distance between **y** and **s.**

2.2 Boundary face method

In this sub-section, an effective Lagrange approximation method, which is based on the surface element, is presented. The surface element is defined in 2D parametric space of the surface rather than in the physic space or other parametric spaces. Thus the geometric components in the considered boundary integral can be computed directly through a parametric transformation. And this parametric transformation is the same as the mapping scheme in the parametric surface, in other word, the geometric quantities in the integral is exactly obtained. A four-node quadrilateral surface element is taken as an illustration example of the physical variable approximation.

The corresponding shape functions we construct in this implementation is:

$$N_1 = \frac{1}{4}(1+\xi)(1+\eta),\ N_2 = \frac{1}{4}(1-\xi)(1+\eta)$$
$$N_3 = \frac{1}{4}(1-\xi)(1-\eta),\ N_4 = \frac{1}{4}(1+\xi)(1-\eta) \tag{6}$$

It should be emphasise here that, these shape functions are only applied to approximate the physical variables on the boundary surfaces. The geometric quantities keep exact in this implementation, and this is the main difference from the standard boundary element method (BEM).

$$u(x,y,z) = u(u,v) = u(\xi,\eta) = \sum_{k=1}^{N} N_k(\xi,\eta)u_k$$
$$q(x,y,z) = u(u,v) = u(\xi,\eta) = \sum_{k=1}^{N} N_k(\xi,\eta)q_k \tag{7}$$

where $x = x(u, v)$, $y = y(u, v)$ and $z = z(u, v)$, u_k and q_k are the values of temperature and the normal flux on the k^{th} boundary node, respectively. N is the total number of interpolating points.

With this approximation scheme, the BIE equation (2) is discretised into:

$$\sum_{j=1}^{M}\int_{\Gamma_j} u^s(\mathbf{s},\mathbf{y})\sum_{k=1}^{N} N_k(\mathbf{s})q_k d\Gamma(\mathbf{s})$$
$$-\sum_{j=1}^{M}\int_{\Gamma_j} q^s(\mathbf{s},\mathbf{y})\sum_{k=1}^{N}(N_k(\mathbf{s}) - N_k(\mathbf{y}))u_k d\Gamma(\mathbf{s}) = 0 \tag{8}$$

After we collocate the field point at every interpolation point, we will get the following system:

$$\mathbf{Gq} - \mathbf{Hu} = 0 \tag{9}$$

in which

$$H_{ik} = \sum_{j=1}^{M}\int_{\Gamma_j} q^s(\mathbf{s},\mathbf{y}_i)(N_k(\mathbf{s}) - N_k(\mathbf{y}_i))d\Gamma(\mathbf{s}) \tag{10}$$

and

$$G_{ik} = \sum_{j=1}^{M}\int_{\Gamma_j} u^s(\mathbf{s},\mathbf{y}_i)N_k(\mathbf{s})d\Gamma(\mathbf{s}) \tag{11}$$

3 BOUNDARY FACE METHOD FOR 3D TRANSIENT HEAT CONDUCTION

In this section, we apply time-domain BIE to solve the transient heat conduction problem. In the transient heat conduction problem, the governing equation is actually the parabolic equation, which consists of four variables considering the temporal variable. Up to three dimensional integral in the time domain BIE is required. There are two way to solve this BIE. The one calculates the three dimensional spatial integral. The other one concerns one dimensional time integral, which is called time convolution, and the integral over the boundary of the considered domain. Since no domain integral is required, we apply the latter method in this paper to analyse the transient heat conduction on one block with several holes.

3.1 Boundary integral equation for 3D transient heat conduction

For the sake of simplification, we will briefly introduce the time-domain BIE for transient heat conduction. More details on this subject can be found in Ma et al (2008) and Dargush & Banerjee (1991).

We first concern the governing equation:

$$\begin{aligned} &k\nabla^2 u = \rho c\dot{u}, && \forall x \in \Omega \\ &u = \bar{u}, && \forall x \in \Gamma_u \\ &-k\frac{\partial u}{\partial n} \equiv q = \bar{q}, && \forall x \in \Gamma_q \\ &u(t_0) = 0, && \forall x \in \Omega \end{aligned} \tag{12}$$

k, ρ and c stand for conductivity, density and specific heat, respectively. Dot means the time derivative. t_0 is the initial time.

We employ the time-dependent fundamental solution to convert equation (12) into the following BIE:

$$c(y)u(y,t) + K\int_{t_0}^{t}\int_{\Gamma} u(x,\tau)q^*(y,x,t,\tau)d\Gamma(x)d\tau$$
$$= K\int_{t_0}^{t}\int_{\Gamma} q(x,\tau)u^*(y,x,t,\tau)d\Gamma(x)d\tau \tag{13}$$

in which $K = k/c\rho$ is the diffusion coefficient,

$$c(y) = \begin{cases} 0 & y \notin \Omega, y \notin \Gamma \\ 1 & y \in \Omega \\ \dfrac{\theta}{2\pi} & y \notin \Gamma \end{cases} \tag{14}$$

and

$$\begin{cases} u^*(y,x,t,\tau) = \dfrac{1}{(4\pi K(t-\tau))^{1.5}} e^{-\frac{r^2}{4K(t-\tau)}} \\ q^*(y,x,t,\tau) = \dfrac{1}{(4\pi K(t-\tau))^{1.5}} \dfrac{2kr}{4K(t-\tau)} e^{-\frac{r^2}{4K(t-\tau)}} \dfrac{\partial r}{\partial n(x)} \end{cases} \tag{15}$$

3.2 Time stepping convolution method

The time interval $[t_0, t]$ can be divided into M increments of duration Δt. We assume that within each increment, the physical variables remain constant with respect to time. Then equation (13) has a time discretised form:

$$c(y)u(y,t) + K\sum_{m=1}^{M}\int_{\Gamma} u(x,t_m)\int_{t_{m-1}}^{t_m} q^*(y,x,t,\tau)d\tau d\Gamma(x)$$
$$= K\sum_{m=1}^{M}\int_{\Gamma} q(x,t_m)\int_{t_{m-1}}^{t_m} u^*(y,x,t,\tau)d\tau d\Gamma(x) \tag{16}$$

in which $t_m = t_0 + m\Delta t$. Letting

$$T^{M+1-m}(y,x,t) = \int_{t_{m-1}}^{t_m} u^*(y,x,t,\tau)d\tau \tag{17}$$

and

$$F^{M+1-m}(y,x,t) = \int_{t_{m-1}}^{t_m} q^*(y,x,t,\tau)d\tau \tag{18}$$

Equation (16) can be simplified into:

$$\begin{aligned} &c(y)u(y,t) + K\sum_{m=1}^{M}\int_{\Gamma} u(x,t_m)F^{M+1-m}(y,x,t)d\Gamma(x) \\ &= K\sum_{m=1}^{M}\int_{\Gamma} q(x,t_m)T^{M+1-m}(y,x,t)d\Gamma(x) \end{aligned} \tag{19}$$

It is obvious that there is no domain integral term appearing in equation (19).

4 NUMERICAL EXAMPLES

4.1 Steady heat conduction analysis in a real dam

To validate our method, a real dam model shown in figure 2(a) is considered as the first example. We do the analysis by three steps: first identify the interfaces between, then discretise the surfaces of all blocks with triangle and quadrilateral elements, and finally, solve the steady-state heat conduction with proper boundary conditions. All the three steps are conducted automatically. The identified interfaces are shown in figure 2(b), and the boundary mesh consisting of 1984 triangle elements and 148 quadrilateral elements is shown in figure 2(c). The heat conductivities of the material for all blocks are taken as 1.0 W/mK. In order to assess the accuracy of the BFM, the following two analytical fields are used as temperature boundary conditions that are imposed on all boundary surfaces.

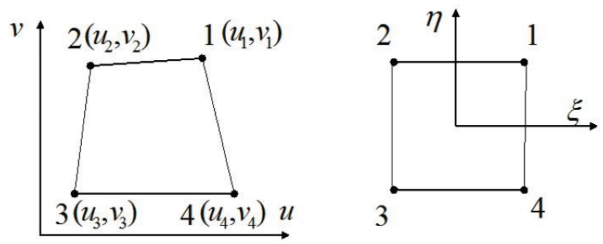

Figure 1: Four-node surface element and coordinate mapping.

$$u = x \tag{20}$$

$$u = x^3 + y^3 + z^3 - 3yx^2 - 3xz^2 - 3zy^2 \tag{21}$$

Very high accuracy has been achieved. The errors of normal fluxes on all surfaces for the linear and cubic fields are 0.034% and 0.071%. The normal fluxes on boundary surfaces for the linear field (equation (20)) are shown in figure 3. And the flux components for the cubic field (equation (21)) are shown in figure 4.

4.2 Transient heat conduction analysis on a block with 8 holes

The second example involves two kinds of special elements. The first one is the triangular element with negative parts which is defined in the mesh around holes. With this type of element, the holes are decomposed into several parts. The element can be described as in figure 5.

In the integration scheme of our implementation, we calculate the integral over the whole triangle element at first, and then the integral over the negative part of the element is subtracted from the total integral. With this element, we can save a lot of boundary grids to represent the tiny holes inside the surface.

The other type of elements is the tube element which is defined on the surfaces of tube shape. The element can be described as in figure 6.

In these elements, the temperature and flux in circumferential direction of the tube are represented in terms of a circular shape function together with a linear or quadratic variation in the longitudinal direction. The circular shape functions are:

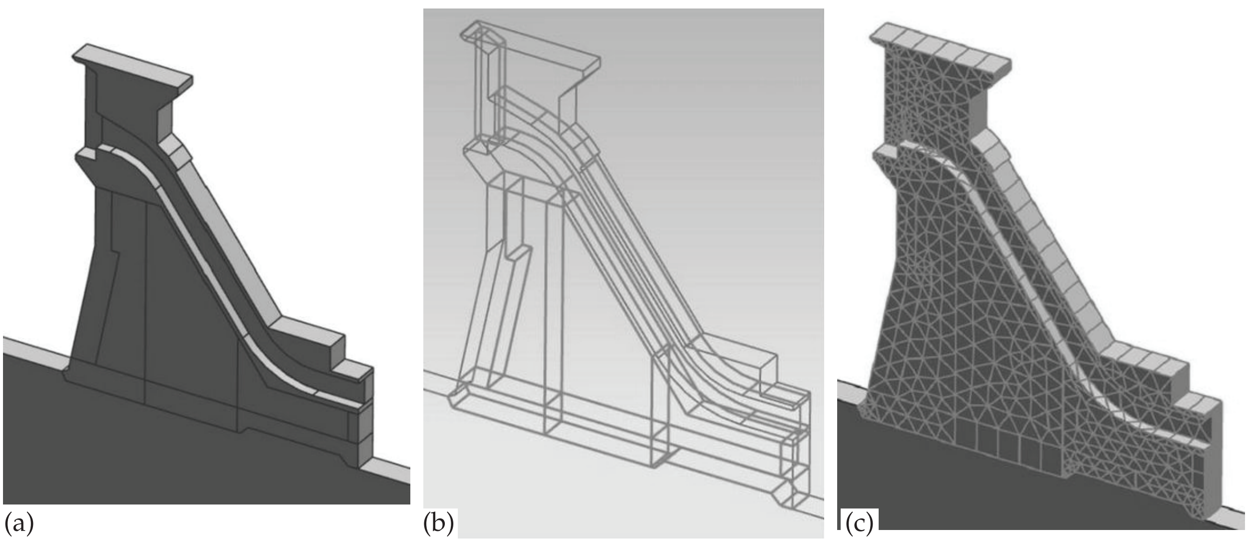

Figure 2: The dam model – (a) CAD model, (b) identified interfaces, and (c) boundary meshes.

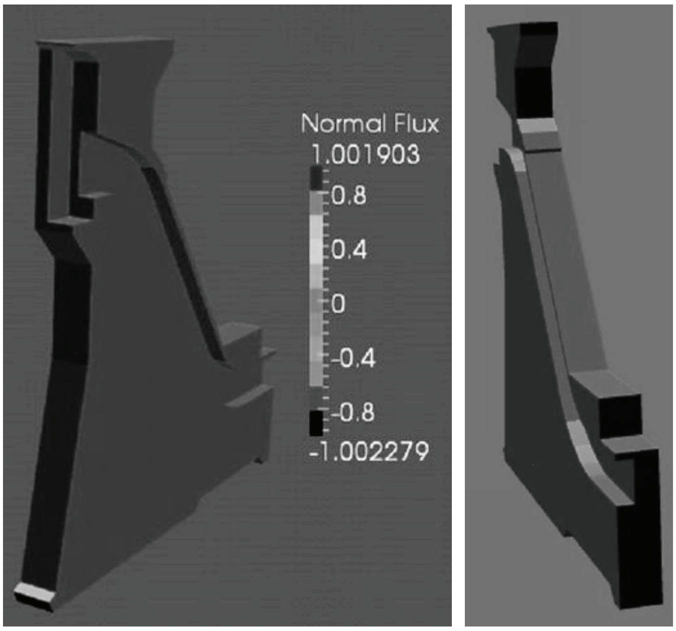

Figure 3: Normal fluxes on surfaces for the linear field.

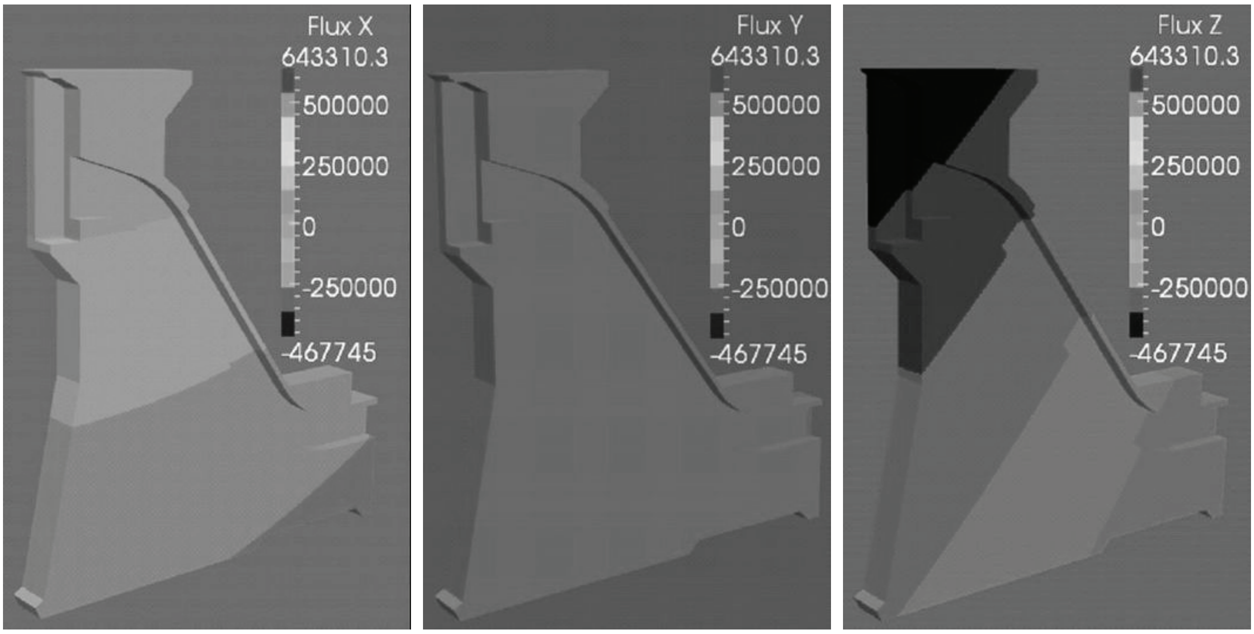

Figure 4: Flux components for the cubic field.

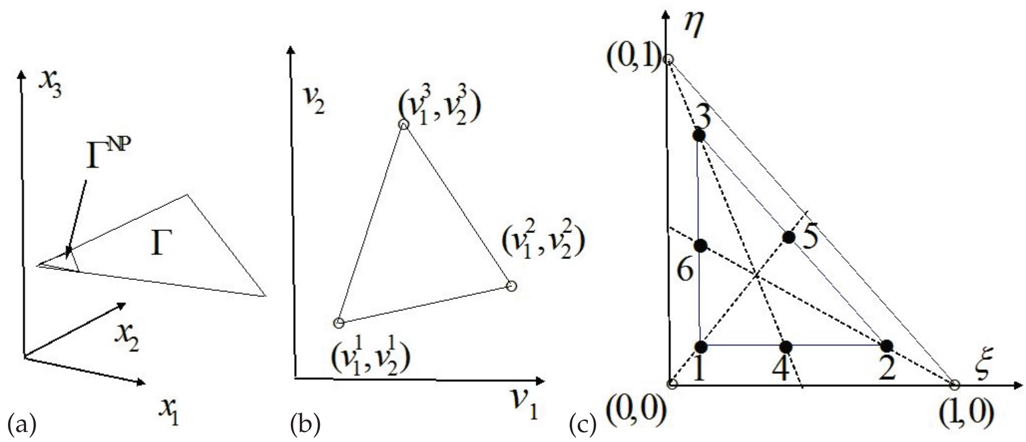

Figure 5: A triangular element with negative parts – (a) element in physical space, (b) element in parametric space of the surface, and (c) element mapped into a local coordinate system.

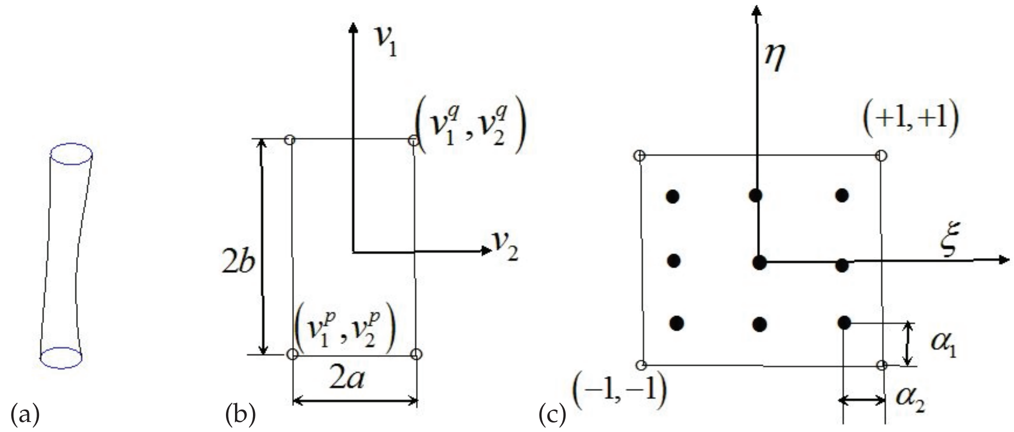

Figure 6: A tube element (o denotes an element vertex and • denotes an interpolating node) – (a) element in physical space, (b) element in parametric space of the surface, and (c) element mapped into a local normalised coordinate system.

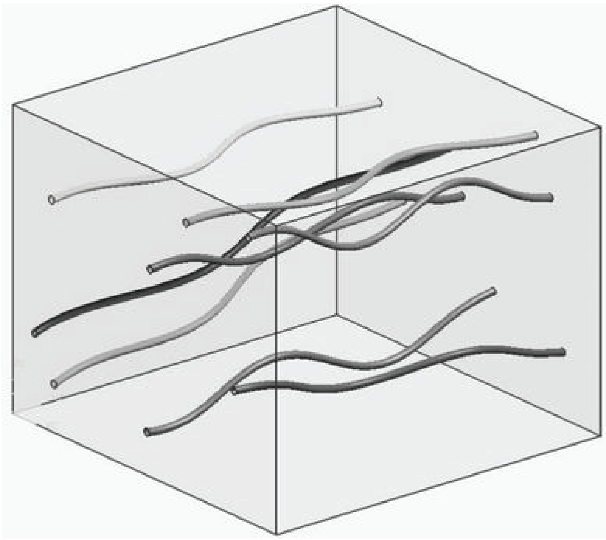

Figure 7: Illustration of the block with 8 free shaped cavities.

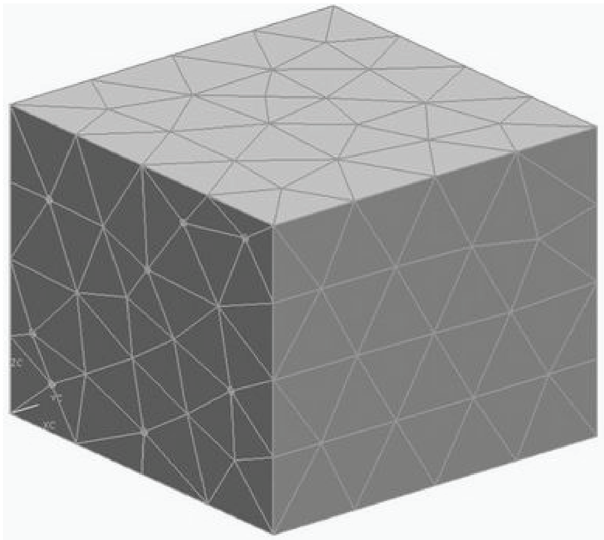

Figure 8: The boundary elements with 8 free shaped cavities.

$$M^0(v_1) = \frac{1}{3} + \frac{2}{3}\cos v_1 \quad (22)$$

$$M^1(v_1) = \frac{1}{3} + \frac{\sqrt{3}}{3}\sin v_1 - \frac{1}{3}\cos v_1 \quad (23)$$

$$M^2(v_1) = \frac{1}{3} - \frac{\sqrt{3}}{3}\sin v_1 - \frac{1}{3}\cos v_1 \quad (24)$$

in which the following linear transformation is applied.

$$v_1 = \pi(\xi + 1.0) - \pi/3 \quad (25)$$

In this example, we generalise the method for structures with free shaped cavities. We concern a block [0, 10]×[0, 10]×[0, 8] with 8 free shape cavities in it as shown in figure 7. The density, conductivity and specific heat of this block are 8.0, 0.4 and 0.4, respectively.

The constant temperature 50 is imposed on the boundary surface $x = 10$. On the free shaped cavities, a constant temperature 10 is imposed.

Table 1: Steps and increments in the second example.

Step number	1	2
Time interval	[0, 10]	[10, 100]
Length of increment	1	5

Other boundary surfaces are insulted. We concern temperature variation history in 100 seconds, and 2 time steps with increments as shown in table 1 are applied in this example. [0, 10] and [10, 100] are applied for this time interval. Within these 2 steps, 10 and 18 increments are applied.

In the implementation of BFM, 1656 nodes and 268 quadratic elements, which consist of 18 tube elements, 66 triangular elements with negative part and 184 regular triangular elements, are employed, which are shown in figure 8.

It is worth noting that on the block surface, which the cavity surface intersects with, there are several

Figure 9: Triangular elements with more than one negative part.

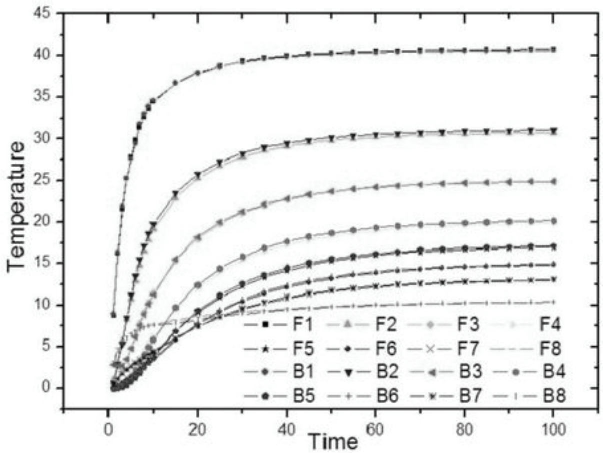

Figure 10: Temperature variation history provided by two methods.

Table 2: Coordinates of sample points in the third example.

1	2	3	4
(9.38, 10, 3.991)	(8.626, 10, 4.167)	(8.059, 10, 4.222)	(7.372, 10, 4.29)
5	6	7	8
(6.783, 10, 4.386)	(6.21, 10, 4.416)	(5.637, 10, 4.542)	(4.42, 10, 4.701)

triangular elements with 2 negative parts as shown in figure 9. Our proposed element still validates in this case.

In the FEM implementation, however, 87231 quadratic tetrahedral elements within the total 129594 nodes are applied to perform this transient analysis. We also compare the results provided by two methods. In this application, we simulate the temperature variation history on 8 sample points which are also 8 nodes in FEM implementation. The coordinates of the sample points are listed in table 2. Results are compared in figure 10.

In figure 10, F stands for the result provided by the FEM and B stands for the result provided by the BFM. The number following F or B stands for the index of the sample point. In this figure, the history computed by our method coincides nicely with that by FEM. This example demonstrates the validity and efficiency of the BFM for transient heat conduction problem on structures with free shaped cavities.

5 CONCLUSION AND FUTURE WORK

The steady heat conduction problem on a dam body which consists of 13 blocks is solved by the BFM. The solution demonstrated that the BFM is very suitable for thermal analysis in dam construction. In the transient analysis, we considered the cooling pipes that lay inside the dam body. Comparison with the FEM implementation illustrates the efficiency of the method.

The results presented in this paper are just a preliminary part of an ongoing research project. More advanced results of transient heat conduction problem on the real concrete massive dam model, which consists of up to 300 hundreds layers and takes into account the varying material properties such as the heat conductivity, heat capacity, hydration heat, and boundary conditions such as ambient temperature, and convection and radiation surface coefficient, will be reported in the near future.

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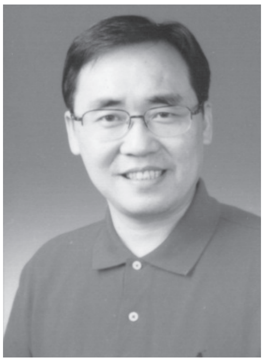

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